Sample Question Paper for Engineering Mathematics-3

1. Find Laplace transform of $\frac{\cos \sqrt{t}}{\sqrt{t}}$ given that $L\{\sin \sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-(1/4s)}$

2. Calculate Spearman's rank correlation coefficient for the following data:

X	32	55	49	60	43	37	43	49	10	20
Y	40	30	70	20	30	50	72	60	45	25

3. Find inverse Laplace transform of $\frac{2s-1}{s^2+8s+29}$

4. If $f(z) = qx^2y + 2x^2 + ry^3 - 2y^2 - 1(px^3 - 4xy - 3xy^2)$ is analytic, find the values of p, q, and r

5. Find Laplace transform of $e^{3t}f(t)$ where $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$

6. Two unbiased dice are thrown. If X represents sum of the numbers on the two dice. Write probability distribution of the random variable X and find mean, standard deviation, and $P([X - 7 \geq 3)$

7. Obtain Fourier series for f(x) = xsin x in the interval $0 \leq x \leq 2\pi$.

8. Using Milne-Thompson's method construct an analytic function

   $f(z) = u + \text{iv}$ terms of z where $u + v = e^x(\cos y + \sin y) + \frac{x-y}{x^2+y^2}$

9. Using convolution theorem find the inverse Laplace transform of $\frac{(s+3)^2}{(s^2+6s+5)^2}$

10. Fit a parabola $y = a + bx + cx^2$ to the following data and estimate y when x = 10

x	1	2	3	4	5	6	7	8	9
y	2	6	7	18	10	11	11	10	9

11. Find Laplace transform of $e^{-(1/2)t}tf(3t)$ if $L\{f(t)\} = \frac{1}{s\sqrt{s+1}}$

12. Find half range sine series for $f(x) = x - x^2, 0 < x < 1$.
   Hence deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} = \frac{\pi^3}{32}$

13. Given regression lines $6y = 5x + 90, 15x = 8y + 130, \sigma_x{}^2 = 16$.
    Find i) $\bar{x}$ and $\bar{y}$, ii) r iii) σ_y^2 and iv) angle between the regression lines.

14. Can the function $a = r + \frac{a^2}{r} \cos \theta$ be considered as real or imaginary part of an analytic function? If yes, find the corresponding analytic function.

15. An unbiased coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails, find moment generating function of X and hence obtain the first moment about origin and the second moment about mean.

16. Evaluate $\int_0^\infty e^{-2t} \cosh t \int_0^t u^2 \sinh u \cosh u \, du \, dt$

17. Find inverse Laplace transform of $\frac{4}{(s-2)^4(s+3)}$ using method of partial fractions.

18. If a continuous random variable X has the following probability density function

19. Find half range cosine series for $f(x) = x, 0 \leqslant x < 2$.
    i) $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots = \frac{\pi^4}{90}$